

# Word Problems

Do word problems look like this to you?

Mary bought # gloves for #####  
each and a belt for ####.  
How much did she spend?

blah blah 4 blah blah \$15.95 blah  
blah blah blah blah \$8.95.  
blah blah blah blah blah?

You see a bunch of numbers and a bunch of words, but you can't relate the two.

## Reading Comprehension

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(If you said \$8.95, you are just scanning for numbers and guessing,  
not reading.)

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(If you said \$8.95, you are just scanning for numbers and guessing,  
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How much did she pay for the belt? **\$8.95**

**So how much did she spend?**

## Translating Shopping into Math

Multiply the number of items purchased by the price per item, then sum these to get the total.

Item	Number	Price	Subtotal
gloves	4	15.95	63.80
belts	1	8.95	8.95
Total			71.75

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Item	Number	Price	Subtotal
gloves	4	15.95	63.80
belts	1	8.95	8.95
Total			71.75

(stuff not covered yet)

Discounts can be applied to prices, subtotals or totals.

Taxes are always applied last and rounded up.

# Translating English into Math

one plus one

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$$1 + 1$$

## Translating English into Math

one plus one

$1 + 1$

three minus two

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one plus one

$1 + 1$

three minus two

$3 - 2$

## Translating English into Math

one plus one

$1 + 1$

three minus two

$3 - 2$

four times seven

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one plus one                       $1 + 1$

three minus two                   $3 - 2$

four times seven                   $4 \times 7$

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four times seven                 $4 \times 7$

twenty divided by five

## Translating English into Math

one plus one  $1 + 1$

three minus two  $3 - 2$

four times seven  $4 \times 7$

twenty divided by five  $20/5$

For **sum**, **product**, **difference**, **quotient** and **ratio**, the word tells you what to do and “and” tells you where to put it:

The **sum** of 4 **and** 7

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The **product** of 4 **and** 3

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The **quotient** of 35 **and** 7

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## Complexity and Ambiguity

The word also tells you where to put the parenthesis.

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Recognize this one?

The sum of the squares of the legs of a right triangle equals the square of the hypotenuse.

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### Recognize this one?

The sum of the squares of the legs of a right triangle equals the square of the hypotenuse.

Let  $a$  and  $b$  be the legs of a right triangle and let  $c$  be the hypotenuse.

$$a^2 + b^2 = c^2$$

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The quotient of  $x$  and the sum of  $x$  and 5.

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$$(x + 3)(x - 4)$$

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Lastly, double check to make sure you answered the question asked, which may not be the variable solved.

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The unknown quantity is the time required. The formula for this type of problem is

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Note the use of “per” in “miles per hour.” This almost always indicates that the quantity is to be multiplied by the corresponding variable.

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The perimeter for a rectangle is given by

$$P = 2w + 2\ell$$

where  $P$  is the perimeter,  $w$  is the width and  $\ell$  is the length. In this case we have

$$120 = 2(50) + 2\ell$$

and can solve for the length,  $\ell$ .

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which has the solution  $x = 3$ .

However the question was “what are the lengths?” so substituting  $x = 3$  gives us either

$$3x - 1 = 3 \times 3 - 1 = 8$$

or

$$2x + 2 = 2 \times 3 + 2 = 8$$

## Pythagorean Theorem

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If the two legs (sides) are given by  $a$  and  $b$  and the hypotenuse (diagonal) by  $c$ , then

$$a^2 + b^2 = c^2 \quad \text{and therefore} \quad c = \sqrt{a^2 + b^2} \quad .$$

Substituting  $a = 300$  and  $b = 400$  gives us  $c = 500$ .

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Substituting  $a = 300$  and  $b = 400$  gives us  $c = 500$ .

However, the question was “how much distance would one save?”.

Going the long way around takes  $300 + 400 = 700$  feet and cutting across takes 500 feet, so you save  $700 - 500 = 200$  feet.

## Red Herrings

Bob is a 50 year old man who lives at 447 Orange Ave. He drives a 4 year old Honda Civic that gets 30 miles per gallon. He pays \$40 a month for telephone service, and \$3000 a year in property taxes. So how many gallons of gas will he need to drive 300 miles?

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Of all the numbers included, which have anything to do with the gas usage?

30 mpg

300 miles

The formula for gas usage is

$$\text{distance in miles} = \left( \frac{\text{miles}}{\text{gallon}} \right) \times \text{gallons}$$

which in this case means

$$300 = 30g$$

where  $g$  is the unknown number of gallons needed.

The purpose of math is to solve real world problems.

Being able to translate real world problems into math is the key to using math effectively.